Waves, Particles, and Icebreakers

Raphael Zumbrunn Sitting here near the ocean, at the border where land meets the sea. I look down at the crashing and churning waves. The air is clean and slightly salty.– What a beautiful day! I take joy in observing nature unfold. I bask in its beautiful complexity, which I find so puzzling and charming at the same time. What I love most is looking out at the different patterns created by the twisting waves. Depending on whether I focus on a single spot or "zoom-out" I get a completely new perspective.

In this essay, I want to talk to you about differing perspectives on waves, how we can understand them and how they can explain why making introductions is so hard sometimes.

Do You Sea What I Sea?

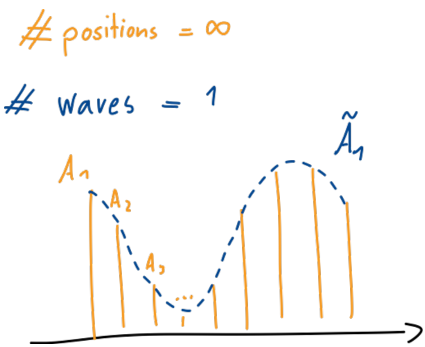
Looking out at the ocean, I see that there is some pattern to the ripples in the water. It seems, from my vantage, that the waves are rolling in from the horizon, almost perfect sinusoids in one direction and almost perfect wavefronts in the other.

Seeing this beautiful mathematical miracle unfold, I turn to my friend sitting next to me eating an apple, and start explaining my epiphany. Instead of sharing my enthusiasm, they decide to play dumb: "Where exactly are those perfect sinusoids you are talking about? Are they with us in the room right now?" I sigh. "Look at the water!", I exclaim, pointing my finger in the general direction of the ocean. "Sure, but where exactly? The patch of water you are pointing to just looks like it's moving up and down." Of course, they know what I mean, but they do bring up a valid point. I can't really point out where the waves are. I mean, of course I can point out a specific wavefront, or point in the general direction, but fundamentally I can't give an exact position of the sinusoid itself. And this makes sense when you think about it, why would you be able to pinpoint the exact position of an object that extends along the entire coast and out into the ocean? It's not like the wave even has a well-defined position.

Describing Waves Is Easy, Actually

Let's take a step back from reality and put on our science goggles (quote due to Prof. Wallny). If I wanted to quantify a wave to its best extent, how would I go about that? You might already have noticed in my previous paragraph that there is a way to describe which wave we are talking about, namely by describing its wavelength and direction. Fundamentally, it is the wavelength that differentiates the ripples from the waves, and the waves from the tides. So, while a wave might not have a perfect positional description, it does have a perfect wavelength description.[1]

When I say a perfect description, I need to clarify what exactly I mean. It is not that I cannot describe a wave in other ways. I could, for example, specify the amplitude of the water at every point of the ocean, and end up with the same informational content as the wavelength description. You will, however, notice that the number of parameters in those two scenarios are different. For the wavelength description, it is sufficient to give a handful of representative wavelengths, whereas the amplitude description requires an almost infinite number of parameters.



When I say that a description is perfect, I mean that it is in some sense simple, meaning it is characterized by only a few parameters. Mathematicians call these differing descriptions or perspectives of the same fundamental object bases. A basis is a collection of fundamental objects that you can mix and match to create a full object. In our case, the description in terms of amplitude per position is the position basis. I am basically mixing and matching how much every point contributes to the whole wave – by setting its amplitude.

The wave description takes the wave itself as its fundamental object. By using these extended objects, we can describe the entirety of the ocean using only a few of these wave objects. It would be reasonable to call this the wave basis, but alas, physicists are not reasonable, which is why we call it the momentum basis.

The Momentum Basis

It is not fully illogical to call this new basis the momentum basis. The reason physicists can get away with naming it the momentum basis is because there is a fundamental relation between wavelength and momentum. This was first discovered for light waves by Nichols and Hull, and by Lebedev, who managed to show that light exerts pressure on objects it gets reflected from. This implies that light waves have some sort of momentum. Moreover, the relation of energy in light and the amount of momentum exerted by it implies that shorter wavelengths carry more momentum.

Using this link, we can go from describing waves using wavelengths to describing them using momenta. Thus, it is not fully unreasonable to call it the momentum basis.

[1] Technically, the perfect description is a wave vector, but that's beside the point.

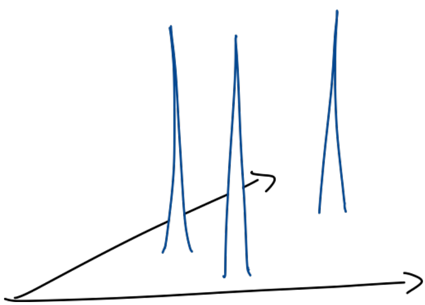
In the previous paragraphs, we found that a wave has no position, but is there also a thing that has no momentum?

Dubious Dirac Deltas

Well, let's consider a completely flat ocean. No breeze is perturbing the surface, no ripples disturb the watery mirror. Now this ocean is the most boring ocean you can imagine because all its amplitudes are zero. It is simple in both the position and the momentum basis. Now we could add a sine wave on top to get the previously mentioned momentum state, but let's try something different for once. What if we just force the position basis to be perfect, what would that look like?

Well, previously, we found that in order for the momentum basis to be perfect, we had to have just a handful of wavelengths that have non-zero amplitude. If we try this for the position, this would mean that only a couple of positions have non-zero amplitude.

Pictorially, this would be just a few spikes of water that would be infinitely thin, poking out of the ocean.

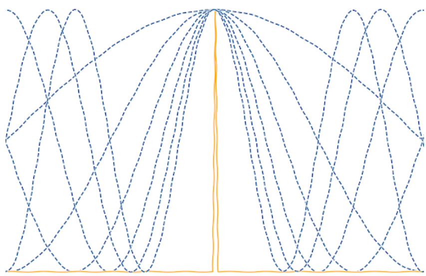


To simplify the logic, let's focus on a single water spike. This spike has, by construction, only one well-defined position. Now what is the momentum of this spike?

We gathered before that we can figure out the momentum or momenta of an ocean by looking at its momentum parts, meaning we need to construct our current state (the spike) from momentum states (sinusoids).

The way to do this rigorously is by using the Fourier transform, which allows us to go from descriptions in waves to descriptions in position. For this simple case, we can also try to intuit how to build the spike. To do this, we first note that the spike has two important properties:

1. It is very sharp → We need very high frequencies (because low frequencies are too soft)
2. It is very localized → We need very low frequencies (to make sure that there is only one peak and not many)



Using this picture, we can see that we need both very high and very low momentum states to build the position state. This means that the momentum is not very well-defined.

This is the inverse of a momentum state with no position. A position state with no defined momentum!

Fundamentally Uncertain

Where does this leave us? We have waves that have no position, and we have spikes that have no momentum. If you are even slightly familiar with quantum mechanics, your alarm bells have been ringing for the last two paragraphs.

And you, my observant reader, are of course 100 % correct! The thing that we just discovered about waves is the so-called uncertainty principle! Specifically, position-momentum uncertainty.

Moreover, there is a pretty nice interpretation to what we just "discovered". I have been calling the sinusoids "waves" for this entire essay, and this is also the name physicists use, but for the second part of the recipe I mislabeled our ingredients. If you think about it for a bit,a better way to describe the position state, the spike, would be a particle!

So not only have we found the position-momentum uncertainty, we found the wave particle duality underpinning some of the most confusing parts of modern physics.

Lies and Deception!

"Hey! Why isn't the essay over yet?", you ask with exasperation. Well, there is an easy answer to this question …I lied. I pretended that there is a direct analogy between water waves and quantum mechanical waves. This claim was not only unsubstantiated, but also wrong! The crucial detail that I withheld from you when exclaiming the uncertainty principle was the following: You can actually know both the position and momentum of a water wave. Or more precisely, you can switch between the two interpretations on command.

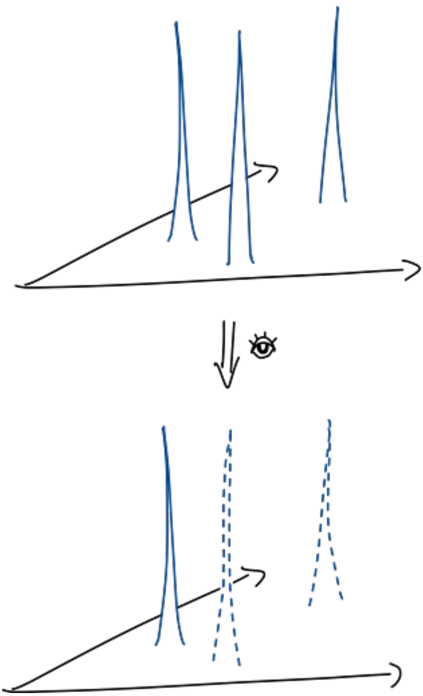
What I mean by this is that we can actually know the position and momentum of the wave perfectly. Or maybe better the position***s*** and momenta***a***. Because in this picture, a sinusoid has a single momentum, but many positions. A spike has many momenta but only a single position. There might be an in-between-object that has some momenta and some positions, but fundamentally we *can* know it all.

So where does uncertainty *really* come from?

The answer is **quantization**.

Or in easier words, uncertainty comes from the fact that if I talk about a particle, it has to have just one position. For a wave, it is easy to say that it can have multiple non-zero amplitudes, but for a particle …As long as I believe in the conservation of particles, I need to also say that a particle may only have one position. The particle number therefore is quantized. I need to have full particles.

So, even if my position basis state is a combination of multiple spikes, the final measurement result of a position will only be one spike.



This so-called wave-function collapse is one of the most puzzling properties of quantum mechanics and is to this day not completely understood.

Incompatibility

So we found that the fact that momentum and position basis were different perspectives are not the fundamental cause for uncertainty. The real reason for uncertainty is the quantization of the number of particles. Or phrased differently, the reason we care about incompatible perspectives (in quantum mechanics we would say "non-commuting operators") is not because it is hard to switch between them, but rather because after switching between them, we also need to choose a single state in the new basis as well.

There are two points to note here:

1. Just because there exist multiple perspectives to a problem, does not mean there will be an uncertainty relation. (I'll explain later why that is)
2. The different perspectives might not be the fundamental reason for uncertainty, but they are a necessary part of it.

One example of compatible perspectives is the position in the x direction and the position in the y direction. While those are still changes in perspective, they are a specifically simple type of change. The main feature that makes them simple is that the number of basis states stays the same. In the wave/position example, building a wave always involves many positions and vice versa. In the pos-x and pos-y case this is no longer so. No matter where I look, simple descriptions stay simple.

The degree to which simple descriptions are not simple in another basis can be measured using the commutator. It is essentially a means to see how compatible two descriptions of a scenario are with each other. For the x and y position, we can find that the commutator $[x, y] = 0$. Which means they are not incompatible → compatible. For the position and momentum, however, we find $[x, p] = {}^{i}/_{\hbar}$. Which means they are incompatible.

To wrap up, incompatibility of bases becomes an issue in quantum mechanics because measurements in quantum mechanics are always forced to be simple (one result only). We can derive how incompatible two sets of bases are using the commutator.

If the Only Tool You Have Is a Hammer . . .

I hope, after having gone through this pretty dense physics text, you will allow me a little bit of silliness. A bit of experimental exploration. As an (almost) fully-trained physical chemist, I should be pretty familiar with quantum physics at this point. And as the old adage goes: "If the only tool you have is a hammer, everything is a nail." Or in this case, if the only thing you know is quantum mechanics, everything is uncertain.

I will extend the formalism of quantum mechanics slightly[2] and try to apply its general vibe to a non-quantum situation and see what happens. There we go:

Sub Essay: Why Making Quick Introductions is Hard

I often struggle with quick introductions, be it to friends of friends or to a group of people during those annoying icebreaker activities.

How am I supposed to break down my 23 years of lived experience into the short and snappy "Hi, my name is Raphael, I like _ and do _" format? It feels to me as if there is some fundamental limit to how I can express myself in this short time, almost as if there were some sort of fundamental uncertainty on any introduction I give.

I can describe myself using a few different perspectives, which depending on the environment might be better suited. To make it easy, I'll consider the two perspectives "serious" and "casual". From the serious perspective, I would highlight my career, what I study, and so on . . . From my casual perspective I would talk about my hobbies (writing niche articles about quantum mechanics amongst other things) and what I like to do in my free time, you get the picture.

Now, I claim that these perspectives are actually incompatible. The argument goes as follows: If I introduce myself to a potential employer, but I use too much of my time to talk about my hobbies, I will come off as not serious enough. On the other hand, if I introduce myself to friends of friends and I only talk about my career, they'll think I'm arrogant and boring.

So the two perspectives are actually fundamentally incompatible (given only a short introduction).

[2] Meaning: Take it wildly out of context and deform it into a shape that has nothing to do with physics.

You see the small comment I added at the end. It is actually important that my introduction is time-limited. This mirrors the importance of particle-number quantization in the above scenario.

We get the same picture again: different perspectives + restriction to simplicity → uncertainty relation.

Cool! We now figured out that real life not made up completely realistic non generated scenarios™ can also exhibit uncertainty relations. What can we learn from this?

Honestly, not much . . . It just shows that sometimes you can't be everything at once. It is somewhat reassuring that it isn't really me being bad at icebreakers that make them painful, but rather the fundamental setup of the situation. So, next time you are trapped in a cringe-inducing icebreaker, don't panic, it's not you, it's just uncertainty.